

Pion production in proton-proton collisions in a covariant one boson exchange model*

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Abstract

Motivated by the renewed interest in studying the pion production on nuclei with protons at few GeV incident energies, we investigate the pion production in proton-proton collisions over an energy range of 300 *MeV* to 2 *GeV*. Starting from a realistic one-boson exchange model with parameters fitted to the amplitudes of the elastic nucleon-nucleon scattering, we perform fully covariant calculations for the total, double and triple differential cross-sections of the $p(p, n\pi^+)p$ and $p(p, p\pi^0)p$ reactions. The calculations incorporate the exchange of π, ρ, ω and σ mesons and treat nucleon and delta isobar as intermediate states. We obtain a reasonably good agreement with the experimental data in the entire range of beam energies. The form of the covariant delta propagator, the cut-off parameter for the πNN and $\pi N\Delta$ vertex form factors and the energy dependence of the delta isobar decay width is investigated.

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1 Introduction

The study of pion production in two nucleon collisions dates back to the mid fifties when one of the earliest measurements of the $p(p, n\pi^+)p$ reaction was performed [1]. This was followed by several measurements of this reaction in 1960's and early seventies [2]. These experiments were usually conducted in emulsions or bubble chambers, consequently the data invariably had poor statistics. However, with the advent of accelerators capable of producing intense high quality beams of protons with energies up to a few GeV and sophisticated detecting systems, it became possible to obtain good quality data on the charged as well as neutral pion production in proton-nucleon collisions at several beam energies [3, 4]. Even the complete kinematical measurements of the three particles in the final state can now be performed [5, 6, 7, 8]. Furthermore, with improved polarisation techniques for beams and targets, the spin observables (asymmetries) are also being measured with great accuracy [6, 7]. These measurements provide a very stringent test of the theoretical models of the pion production which is the dominant inelastic channel in nucleon-nucleon (NN) collisions.

A thorough investigation of the pion production in the elementary NN collisions within a fully relativistic model is essential in the context of recent efforts to develop a covariant two nucleon model (TNM) to describe the $A(p, \pi)B$ reactions [9, 10], where several parameters have to be fixed from the study of the former reaction. The precise information about the pion production cross sections in elementary NN collisions is also important for the description of the dynamics of the heavy ion collisions within the kinetic theories (eg. BUU) [11, 12].

Although several models have been proposed to study the $NN \rightarrow NN\pi$ reactions [13], only a few have taken the fully relativistic Feynman diagram approach [14, 15, 16, 17]. However, a systematic study of all the observables (total, single, double, and triple differential cross sections and asymmetries) of $p(p, n\pi^+)p$ and

$p(p, \pi^0)p$ reactions over a wide range of beam energies within the covariant model is still lacking, and the form of several ingredients of the model remains to be far from being determined unambiguously. Moreover, the role of σ and ω mesons have not been explicitly studied in such theories so far.

Fully relativistic calculations could be necessary even at the beam energies closer to the $NN\pi$ threshold, where the non-relativistic models underpredict the experimental data by a factor of approximately 5 [18]. One of the consequences of the relativistic effects is that they lead to an enhancement of the axial vector current of the NN system [19], which can increase the pion production cross sections, and possibly contribute to an explanation of the above discrepancy (see e.g Horowitz et al. [13]) to a great extent.

The aim of this paper is to perform a detailed investigation of the $p(p, n\pi^+)p$ and $p(p, p\pi^0)p$ reactions using a fully relativistic Feynman diagram technique. We carry out our calculations within an effective one-boson exchange (OBE) nucleon-nucleon (N-N) scattering mechanism, which includes both nucleon and delta isobar excitations in the intermediate states. We consider the exchange of π , σ , ρ , and ω mesons. Most of the parameters of the OBE model are determined by fitting to the N-N scattering data [17]. Different delta isobar propagators proposed in the literature [20, 21, 22] have been examined in order to remove the confusion about their most appropriate form. The relative importance of the contributions of different exchanged meson in the energy range 300 MeV to 2.0 GeV has been discussed.

In the next section, we give the details of our model and derive the expressions of various amplitudes. The comparison of our calculations with the experimental data and the discussion of the results are presented in section 3. The summary and conclusions of our work are presented in section 4.

2 The amplitudes of pion production in One Boson exchange model

To obtain the desired production cross section for the $NN \rightarrow NN\pi$ processes, we use a fully covariant method based on an effective one-boson-exchange model which describes at the same time the elastic nucleon-nucleon scattering. Pions are simply produced from the external nucleon lines (see Figs. 1a - 1d) or if possible, also from the internal meson lines (Figs. 1e-1h). The pion production via formation, propagation and the subsequent decay of the delta isobar is included. It may be noted that in Fig. 1 we have not shown the so called 'pre-emission' diagrams. However, in actual calculations their contributions are also included.

2.1 Model Lagrangian

The nucleon-nucleon interaction is described by the exchange of π , σ , ρ and ω mesons in term of which we parametrize the T-matrix. The corresponding Lagrangian densities are given by

$$\mathcal{L}_{\pi NN} = -\frac{f_\pi}{m_\pi} \bar{\Psi}_N \gamma_5 \gamma_\mu \boldsymbol{\tau} \cdot (\partial^\mu \boldsymbol{\Phi}) \Psi_N. \quad (1)$$

$$\mathcal{L}_{\rho NN} = -g_\rho \bar{\Psi}_N \left(\gamma_\mu + \frac{k_\rho}{2m_N} \sigma_{\mu\nu} \partial^\nu \right) \boldsymbol{\tau} \cdot \boldsymbol{\rho}^\mu \Psi_N. \quad (2)$$

$$\mathcal{L}_{\omega NN} = -g_\omega \bar{\Psi}_N \left(\gamma_\mu + \frac{k_\omega}{2m_N} \sigma_{\mu\nu} \partial^\nu \right) \omega^\mu \Psi_N. \quad (3)$$

$$\mathcal{L}_{\sigma NN} = g_\sigma \bar{\Psi}_N \sigma \Psi_N. \quad (4)$$

It may be noted that we have used a pseudovector coupling for the πNN vertex. In addition the couplings of the mesons to the delta resonance are needed. Due to the isospin conservation only ρ and π mesons couple to the delta resonance. The Lagrangian densities for these processes are

$$\mathcal{L}_{\pi N \Delta} = \frac{f_\pi^*}{m_\pi} \bar{\Psi}_\mu \mathbf{T} \cdot \partial^\mu \Phi_\pi \Psi_N + \text{h.c.} \quad (5)$$

$$\mathcal{L}_{\rho N \Delta} = i \frac{g_\rho^*}{m_\Delta + m_N} \bar{\Psi}_\mu \mathbf{T} (\partial^\nu \rho^\mu - \partial^\mu \rho^\nu) \gamma_\nu \gamma_5 \Psi_N + \text{h.c.} \quad (6)$$

To insure the gauge invariance of the $\rho - N$ amplitude we must also include the diagrams 1e - 1h, which depict the processes where the ρ meson decays into two pions in flight. The corresponding Lagrangian density is given by

$$\mathcal{L}_{\rho\pi\pi} = g_{\rho\pi\pi} [(\partial^\mu \Phi) \times \Phi] \cdot \rho_\mu. \quad (7)$$

In Eqs.(1-7), Ψ_N represents the Dirac spinor for the nucleon in the spin-isospin space. Ψ_μ is the corresponding Rarita-Schwinger spinor for the delta isobar. Φ , ρ , σ and ω represent the pion, rho, sigma and omega meson fields, respectively. \mathbf{T} and $\boldsymbol{\tau}$ represent the isospin operator for the transition $\Delta \rightarrow N\pi$ and the isospin Pauli matrices, respectively. We use the conventions and notations of Ref. [23] for definitions of spinors, operators and isospin matrices, respectively.

Since we use the Lagrangians (1-4) to directly model the T-matrix, we have also included a nucleon-nucleon-axial vector-isovector vertex, with the Lagrangian density given by

$$\mathcal{L}_{NNA} = \sqrt{g_A} \bar{\Psi} \gamma_5 \gamma_\mu \boldsymbol{\tau} \Psi \cdot \mathbf{A}^\mu. \quad (8)$$

In Eq. (8) \mathbf{A} represents the axial vector meson field. This term is introduced because in the limit of large axial meson masses (m_A) it cures the unphysical behaviour in the angular distribution of NN scattering caused by the contact term in the one-pion-exchange amplitude [17], if g_A is chosen to be

$$g_A = \frac{1}{3} m_A \left(\frac{f_\pi}{m_\pi} \right)^2. \quad (9)$$

with very large ($\gg m_N$) m_A .

The determination of other coupling constants used in this work is discussed in the next section.

2.2 Coupling constants

The coupling constants appearing in Eqs. (1 - 4) were determined by fitting to the elastic proton-proton and proton-neutron scattering data. In the fitting procedure we also take into account the finite size of the nucleons by introducing the form-factors

$$F_i = \left(\frac{\Lambda_i^2 - m_i^2}{\Lambda_i^2 - q_i^2} \right), i = \pi, \rho, \sigma, \omega, \quad (10)$$

at each interaction vertex, where q is the four momentum and m the mass of the exchanged meson. The form factor suppresses the contributions of high momenta and the parameter Λ , which governs the range of suppression, can be directly related to the hadron size. Since the data in the entire range of beam energies cannot be reproduced with an energy independent set of parameters, we have used the following energy dependence for the coupling constants

$$g(\sqrt{s}) = g_0 \exp(-\ell \sqrt{s}). \quad (11)$$

The parameters (g_0 , Λ , and ℓ) were determined [17] by fitting to the relevant proton-proton and proton-neutron data at three beam energies of 1.73 *GeV*, 2.24 *GeV*, and 3.18 *GeV*, where a good fit to the elastic scattering amplitudes were obtained. Table 1 shows the values of the parameters obtained by this procedure. It may be noted that for the case of pion we have shown in this table the constant g_π which is related to f_π of Eq. (1) as $g_\pi = (f_\pi/m_\pi)2m_N$.

The value of the coupling constant f_π^* for the $\pi N \Delta$ vertex has been determined from the $\Delta \rightarrow N + \pi$ decay and its value is 2.13. As in the case of NN scattering, we use form factors for the $N - \Delta$ vertices as well, which have a dipole form

[16, 17]

$$F_i^* = \left(\frac{\Lambda_i^{*2} - m_i^2}{\Lambda_i^{*2} - q_i^2} \right)^2, i = \pi, \rho. \quad (12)$$

The parameters g_ρ^* , Λ_π^* and Λ_ρ^* were determined by fitting to mass differential cross sections for the reaction $N + N \rightarrow N + \Delta$ in the energy range of 1-2.5 GeV. Their values are,

$$\begin{aligned} \Lambda_\pi^* &= 1.421 \text{ GeV}, \\ \Lambda_\rho^* &= 2.273 \text{ GeV}, \\ g_\rho^* &= 7.4. \end{aligned} \quad (13)$$

For the graphs 1e - 1h, we have taken $g_{\rho\pi\pi} = 2g_\rho$ [24, 25]. In this way the NN meson and $N\Delta$ vertices are determined rather reliably. We then assume that their off-shell dependence is determined solely by the form factors (10) and (12).

For the delta isobar propagator we used the form given by Benmerrouche et al. [22]

$$\begin{aligned} G_{\mu\nu}^\Delta(p) &= -\frac{i(\not{p} + m_\Delta)}{p^2 - m_\Delta^2} \\ &\quad [g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2}{3m_\Delta^2}p_\mu p_\nu + \frac{1}{3m_\Delta^2}(p_\mu\gamma_\nu - p_\nu\gamma_\mu)]. \end{aligned} \quad (14)$$

Similar expression for the Δ propagator has been derived also in Ref. [26]. It should be noted that the form of the propagator for an interacting delta isobar remains the same as in Eq. (14) [22]. In appendix B, we have presented a review of the derivation of this propagator and have discussed the difficulties associated with Δ isobar propagators presented by Williams and other authors [20, 21].

The mass of the delta isobar m_Δ appearing in the denominator term $(p^2 - m_\Delta^2)$ is modified by adding to it an imaginary width, which is a function of the pion-nucleon center of mass momentum \mathbf{p}'_π . This is due to the fact that Δ isobar

is not a stable particle and it decays with a width which varies with its mass [27]. There are several prescriptions for the mass dependent delta isobar width given in the literature [16, 28, 29]. We have performed calculations by using the following forms

$$\Gamma_D(p'_\pi) = \Gamma_0 \left(\frac{p'^3_\pi}{p^{R3}_\pi} \right) \left(\frac{p^{R2}_\pi + \epsilon^2}{p'^2_\pi + \epsilon^2} \right), \quad (15)$$

where

$$p'^2_\pi = \frac{[p_i^2 - (m_N - m_\pi)^2][p_i^2 - (m_N + m_\pi)^2]}{4p_i^2}. \quad (16)$$

In Eq. (16), p_i is the four-momentum of the intermediate delta isobar (see Fig. 1). p^R_π used in Eq. (15) is obtained from Eq. (16) by substituting $p_i^2 = m_\Delta^2$. The constant Γ_0 is the free delta width; its value is taken to be 0.120 GeV and $\epsilon = 0.16$. This form has been used by Dmitriev et al. [28]. Verwest [15] has used a somewhat different form which is given by

$$\Gamma_{ver}(p'_\pi) = \Gamma_0 \left(\frac{p'^3_\pi}{p^{R3}_\pi} \right) \left(\frac{\sqrt{p'^2_\pi + m_\pi^2} + m_N}{2m_N} \right), \quad (17)$$

where Γ_0 has the same value as given above.

In the next subsection we present the expressions for the amplitudes of various graphs as shown in Fig. 1.

2.3 Amplitudes and cross sections

After having established the interaction Lagrangians and the form of the delta isobar propagator, we can now proceed to calculate the amplitudes corresponding to the various diagrams associated with the reactions $p(p, n\pi^+)p$ and $p(p, p\pi^0)p$. The Feynman rules for writing down these amplitudes are well known (see e.g. [30]). The isospin part is treated separately which gives rise to a constant factor for each graph. In table 2 we give the values of these factors for the post-emission (shown in Fig. 1) as well as for the pre-emission graphs corresponding to the

emission of π^+ and π^0 mesons. Values for the pion emission from the intermediate states (Figs. 1e-1h) are shown in table 3. It may be noted that in these tables the values for the π^+ case are calculated by assuming particles 3 and 4 as neutron and proton respectively. Isovector corresponds to the exchange of π and ρ mesons while isoscalar to that of σ and ω mesons.

In the following we give the expressions for the amplitudes for the pion production processes via excitation of both nucleon and delta isobar intermediate states for one graph eg. Fig. 1c. We also give expression for one diagramm (Fig. 1f) where pions are emitted from an intermediate meson. The amplitudes for other similar graphs can be written in a straight forward manner.

(i) Nucleon intermediate state and pion exchange

$$A_N^\pi(c) = -Q_N^\pi(c) \left(\frac{f}{m_\pi} \right)^3 \bar{\psi}(p_3) \gamma_5 \gamma_\lambda q^\lambda \psi(p_1) D_\pi(q) \\ \times \bar{\psi}(p_4) \gamma_5 \gamma_\mu p_\pi^\mu D_N(p_i) \gamma_5 \gamma_\nu q^\nu \psi(p_2), \quad (18)$$

where $D_\pi(q)$ and $D_N(p_i)$ are the propagators for the exchanged pion and the intermediate nucleon respectively, which are defined as

$$D_\pi(q) = \frac{i}{q^2 - m_\pi^2}, \quad (19)$$

$$D_N(p_i) = i \frac{p_{i0} \gamma^0 + m_N}{p_i^2 - m_N^2}. \quad (20)$$

Various momenta appearing in Eq. (18) are defined in Fig. 1. The intermediate momenta are given by $q = p_1 - p_3$ and $p_i = p_\pi + p_4$. ψ is the Dirac spinor in the spin space and $Q_N^\pi(c)$ is the isospin coupling factor for the nucleon pole as shown in Table 2.

(ii) Delta isobar intermediate state and pion exchange

$$A_\Delta^\pi(c) = -Q_\Delta^\pi(c) \left(\frac{f}{m_\pi} \right) \left(\frac{f^*}{m_\pi} \right)^2 \bar{\psi}(p_3) \gamma_5 \gamma_\lambda q^\lambda \psi(p_1) D_\pi(q) \\ \times \bar{\psi}(p_4) p_\pi^\mu G_{\nu\mu}^\Delta(p_i) q^\nu \psi(p_2), \quad (21)$$

where $G_{\mu\nu}^\Delta(p_i)$ is the delta isobar propagator as discussed in the previous subsection, and $Q_\Delta^\pi(c)$ is the isospin coupling factor for the delta pole as shown in Table 2.

(iii) Nucleon intermediate state and ρ meson exchange

$$A_N^\rho(c) = -Q_N^\rho(c) \left(\frac{f}{m_\pi} \right) g_\rho^2 \bar{\psi}(p_3) \left(\gamma_\mu + \frac{ik_\rho}{2m_N} \sigma_{\mu\nu} q^\nu \right) \psi(p_1) D_\rho^{\nu\alpha}(q) \\ \times \bar{\psi}(p_4) \gamma_5 \gamma_\lambda p_\pi^\lambda D_N(p_i) (\gamma_\alpha - \frac{ik_\rho}{2m_N} \sigma_{\alpha\beta} q^\beta) \psi(p_2), \quad (22)$$

where $D_\rho(q)$ is the propagator for the ρ meson defined as

$$D_\rho(q)^{\mu\nu} = -i \left(\frac{g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}}{q^2 - m_\rho^2} \right), \quad (23)$$

with $g^{\mu\nu}$ being the usual metric tensor. In Eq. (22) $\sigma_{\mu\nu}$ is defined as

$$\sigma_{\mu\nu} = \frac{i}{2} (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu), \quad (24)$$

Other quantities remain the same as earlier.

(iv) Delta isobar intermediate state and ρ meson exchange

$$A_\Delta^\rho(c) = -Q_\Delta^\rho(c) \left(\frac{f^*}{m_\pi} \right) g_\rho \frac{g_\rho^*}{m_\Delta + m_N} \bar{\psi}(p_3) \left(\gamma_\alpha + \frac{ik_\rho}{2m_N} \sigma_{\alpha\nu} q^\nu \right) \psi(p_1) \\ \times D_\rho^{\alpha\nu}(q) \bar{\psi}(p_4) p_\pi^s G_{ts}^\Delta(p_i) (q^\mu g^{t\nu} - q^t g^{\mu\nu}) \gamma_\mu \gamma_5 \psi(p_2), \quad (25)$$

In Eq. (25) all the quantities are the same as defined earlier.

(v) Nucleon intermediate state and σ meson exchange

$$A_N^\sigma(c) = -Q_N^\sigma g_\sigma^2 \left(\frac{f_\pi}{m_\pi} \right) \bar{\psi}(p_3) \psi(p_1) D_\sigma(q) \bar{\psi}(p_4) \\ \times \gamma_5 \gamma_\mu p_\pi^\mu D_N(p_i) \psi(p_2), \quad (26)$$

where $D_\sigma(q)$ is the propagator for the sigma meson whose form is the same as that given by Eq. (19) except that in the denominator the pion mass is replaced by that of the sigma meson.

(vi) Nucleon intermediate state and ω meson exchange

In this case the form of the amplitude is the same as that of the ρ meson exchange.

(vii) Pion emission from the decay of ρ meson in the intermediate states (Fig.1f)

$$\begin{aligned}
A^{\rho\pi\pi}(f) = & -ig_\rho g_{\rho\pi\pi} \frac{f_\pi}{m_\pi} \bar{\psi}(p_3)(\gamma_\mu + \frac{ik_\rho}{2m_N} \sigma_{\mu\nu} q^\nu) \psi(p_1) D_\rho^{\nu\mu} p_{\pi\nu} \\
& \times D_\pi(q') \bar{\psi}(p_4) \gamma_5 \gamma_\eta q'^\eta \psi(p_2),
\end{aligned} \tag{27}$$

where $q' = p_4 - p_2$.

(viii) Nucleon intermediate state and heavy axial meson exchange

$$\begin{aligned}
A_N^A(c) = & -Q_N^A(c) g_A \left(\frac{f_\pi}{m_\pi} \right) \bar{\psi}(p_3) \gamma_5 \gamma_\mu \psi(p_1) D_A^{\mu\nu}(q) \\
& \times \bar{\psi}(p_4) \gamma_5 \gamma_\mu p_\pi^\mu D_N(p_i) \gamma_5 \gamma_\nu \psi(p_2),
\end{aligned} \tag{28}$$

where $D_A^{\mu\nu}$ is the propagator for the axial vector meson which is defined by

$$D_A^{\mu\nu}(q) = -i \left(\frac{g^{\mu\nu}}{q^2 - m_A^2} \right) \tag{29}$$

In Eq. (29), the mass of the axial meson was taken to be very large (188 GeV), as the corresponding amplitude is that of the contact term.

The amplitudes given above can be simplified by contracting out the gamma matrices using the Dirac equation whenever applicable. The simplified expressions are given in Appendix A. The total amplitude is obtained by summing (coherently) the amplitudes corresponding to all the graphs. It should be noted that the exchange graphs have an extra minus sign due the antisymmetrisation.

The general formula for the invariant cross sections of the $N + N \rightarrow N + N + \pi$ process is written as

$$d\sigma = \frac{m_N^4}{2\sqrt{(p_1 \cdot p_2)^2 - m_N^4}} \frac{1}{(2\pi)^5} \delta^4(P_f - P_i) |A_{fi}|^2 \Pi_{a=1}^3 \frac{d^3\mathbf{p}_a}{E_a}, \tag{30}$$

where A_{fi} represents the sum of all the amplitudes, P_i and P_f the sum of all the momenta in the initial and final states respectively, and p_a the momenta of the three particles in the final state. The corresponding cross sections in the laboratory or center mass systems can be written from Eq. (30) by imposing the relevant conditions.

3 Comparison to data and discussions

The model presented in the previous section has been used to study the available data on the total, double and triple differential cross sections for the $p + p \rightarrow n + \pi^+ + p$ and $p + p \rightarrow p + \pi^0 + p$ reactions for beam energies 300 MeV to 2 GeV. We emphasize here that the parameters described in the previous section have been kept fixed throughout in all the calculations described subsequently.

3.1 Total pion production cross section

In Figs. 2a and 2b, we show a comparison of our calculations with the experimental total cross sections for the reactions $p + p \rightarrow p + n + \pi^+$ and $p + p \rightarrow p + p + \pi^0$ respectively, as a function of beam energy. The dashed lines represent the results obtained by including only those graphs where pion production proceeds via intermediate delta isobar states, while the dashed-dotted lines give the results with only nucleon intermediate states. The solid lines show the results where all the graphs are included. We note that the measured cross sections are reproduced reasonably well by our calculations in the entire range of beam energies. This is remarkable in view of the fact that none of the parameters of the model have been adjusted to the pion production data. Furthermore, they were determined by fitting the NN elastic scattering data (differential cross sections) at beam energies above 1 GeV. The same set of parameters (with an energy dependence of the coupling constants given by Eq. (11)) seems to reproduce the

data at low energies as well.

The contribution of the delta isobar excitation is not important at beam energies below approximately 350 MeV , which is due to the fact that at lower energies pions are predominantly in a relative S -state; thus the possibility of forming a delta isobar is greatly reduced. However, the pion production is dominated by the Δ isobar excitation at higher beam energies.

In Fig. 3, we show the contribution of various meson exchange processes to the total cross section of the $(p, n\pi^+)$ reaction in the considered range of beam energies. The contributions of the heavy axial meson exchange are not shown in this figure as they are negligibly small. We note that the pion exchange graphs dominate the production process for all the energies. The contribution of the ρ meson exchange is almost negligible at lower beam energies, and even in the energy range of 1 - 2 GeV , it is at least an order of magnitude smaller than that of the pion exchange process. Dmitriev et al. [28] have made the similar observation for the reaction $p + p \rightarrow n + \Delta^{++}$ within a similar type of model. This lends a *posteriori* credence to the calculations presented in Ref. [10] for the $A(p, \pi)B$ reaction within a covariant two-nucleon model at a beam energy of 800 MeV , where only one-pion exchange intermediate processes were considered.

At the incident energies very close to the pion production threshold the contributions of the σ and ω meson exchange are as strong as that of the pion exchange. Therefore calculations, which do not include these mesons, underestimate the pion production cross sections near the threshold (see eg. Horowitz et al. in [13], and [31]).

As we note from Table 1, the cut-off parameters Λ must have values in the range of 1.0 - 1.6 GeV in order to fit the NN scattering data. Similar values for this parameter have been used in almost all the earlier calculations[10, 32, 33, 34] of the proton induced pion production (which mostly included only pion exchange

contributions, requiring therefore only Λ_π and Λ_Δ which were taken to be the same). In Fig. 4 we show the total cross sections for the $p(p, n\pi^+)p$ reaction as a function of beam energy for two values of Λ_π . The solid and the long dashed lines show the results obtained with values 1.005 GeV and 0.63 GeV respectively for this parameter. It is apparent that the latter value of Λ_π leads to a smaller cross section and a poorer fit to the data. Smaller values of Λ imply a reduction in the range of the pion exchange contribution to the NN interactions which reduces its contribution to the pion production cross section as this is the dominant process as has been discussed earlier. Although a value of $\Lambda_\pi \leq 0.8$ is consistent with the chiral bag model [35], the quantitative description of the NN data requires a value $\geq 1.0 \text{ GeV}$ [36]. We therefore, stick to the value of Λ_π as shown in Table 1.

In Fig. 5, we investigate the effect of using the pseudoscalar coupling for the πNN vertex on the π^+ production cross section. In this figure we show the ratio of the total cross sections obtained by using the pseudovector (σ_{PV}) and pseudoscalar (σ_{PS}) couplings for the πNN vertex as a function of beam energy. The solid line represents the results obtained by including all the graphs while the long dashed line give the results obtained with only nucleon intermediate excitations. It is clear that the pion production is enhanced if the pseudoscalar coupling is used. This effect is very large at lower beam energies. At higher beam energies, where Δ isobar excitation dominates, this effect still persists even when all the graphs are included in the calculations.

In the one pion exchange picture of the NN interaction, when both nucleon are on the mass shell, it will not be possible to make a difference between the pseudovector and pseudoscalar couplings, as both will produce the same result. However, in Fig. 1, one of the nucleons can go off-shell. Therefore, in this case the two couplings will produce different results. One can show (taking the case of just one diagram, say for example Fig. 1a) that the ratio of the amplitudes

obtained by using pseudovector and pseudoscalar couplings is given approximately by $(E_\pi/2m_N)$. At lower beam energies this ratio is very small, which explains to some extent the trend seen in Fig. 5.

In general, the pseudovector coupling for the πNN vertex is preferred. The p -wave interaction between pions and nucleons is very strong and attractive; the formation of the Δ isobar is the consequence of this interaction. On the other hand, the s -wave interaction is very weak. The pseudovector coupling automatically incorporates these features due to the derivative term. One can show purely on formal grounds that it is also consistent with the constraint imposed by the PCAC hypothesis [19]. The derivative of the axial vector current operator based on the pseudovector πNN coupling vanishes as the pion mass goes to zero. The pseudoscalar coupling does not satisfy this criterion; it also produces a s -wave interaction which is much too strong [37]. Furthermore, this coupling has a history of problems in Dirac calculations [38]. Therefore we do not consider the pseudoscalar couplings for the πNN vertex in our calculations.

3.2 Double differential cross section

We have also calculated the double differential cross sections for the $pp \rightarrow \pi^+ X$ reactions and have compared the results with one set of existing experimental data [39] at the proton incident energy of 800 MeV, in Fig. 6; this set of data were also analysed by Verwest [15]. It is clear that the contributions of the delta isobar excitation (dashed lines) dominate the total cross sections (solid lines), whereas those of the nucleon intermediate state (dotted lines) are very small. The agreement between theory and the data is reasonably good at forward pion angles. However, at higher pion angles our calculations overpredict the data by factors of 1.5 to 2. The peak cross sections are affected by the different multipolarities (dipole or monopole) of the form-factors (Eq. (10) - (11)) and forms of the delta decay width Γ_D (Eq. (15)). For instance, with a monopole

form factor for the the $N\Delta\rho$ vertex, the cross sections near the peak are reduced by approximately 20 %. On the other hand, using the width Γ_{ver} enhances the cross sections near the peak region further by about 5-6 %. There is, therefore, some scope to explain the small disagreement between data and our calculations at larger pion angles. This also indicates that these measurements may be useful in differentiating between various forms of the formfactors and delta decay widths.

The experimental points towards the larger momentum ends of the spectra come from the processes like πd final states which is obviously not included in our calculations.

3.3 Triple differential cross section

We now compare our results with data taken in kinematically complete measurements where all the three particles in the final channels are measured. In Fig. 7, we present a comparison of our calculations with the data [5] for the triple differential cross sections for two sets of pion and proton angles as a function of outgoing proton momentum at the beam energy of 800 MeV . The peak region in this figure corresponds to the final state proton momenta associated with the formation of the delta isobar and its decay. The solid lines (dashed lines) in this figure show the results of calculations which include the contributions of all the graphs (only delta isobar intermediate states). Clearly the peak region is dominated by the delta excitation process. At higher angles and in the regions away from the delta excitation peak, the contributions of nucleon excitation are not negligible as can be seen in the lower part of this figure. The shift in the peak position towards higher momenta at larger angles is due to the fact that at larger pion angles the delta isobar formation is associated with larger outgoing pion momenta.

It should, however, be noted that the position of the peak in the calculated cross sections is shifted towards somewhat larger momenta as compared to the

experimental data. Obviously the form of the delta decay width will play a crucial role in determining the peak position. In Fig. 7 we have used the decay width Γ_{ver} . The effect of using other forms of the decay width is shown in Fig. 8, where the solid lines represent the results of the calculations performed with the delta decay width of Verwest, while the dashed lines corresponds to that of Dmitriev. The short dashed lines show the results obtained with Dmitriev's form but with a value of $\Gamma_0 = 0.100 \text{ GeV}$ [5]. We see that the peak positions of the experimental cross sections are correctly reproduced by calculations using the Decay width of Dmitriev. However, the absolute magnitudes of the cross sections are a bit too high at larger pion angles.

4 Summary and conclusions

In this paper we have presented a fully covariant one-boson exchange model to describe the $p(p, n\pi^+)p$ and $p(p, p\pi^0)p$ reactions in the beam energy (E_{beam}) range of 300 MeV to 2 GeV. The model contains only the physical parameters (like coupling constants and cutoff masses), which were determined by fitting to the NN elastic scattering data at three beam energies above 1 GeV. All the parameters so determined were held fixed throughout the considered energy range; none of the parameters were adjusted to the pion production data of any kind. We considered the exchange of the π , ρ , σ and ω mesons; the latter two were not considered in earlier calculations of the $NN\pi$ processes. The calculations included pre- and post-emission graphs and considered the excitation of both the delta isobar and nucleon intermediate states. The decay of the ρ meson in flight (the so-called intermediate emission diagram) was also incorporated in our calculations.

We found that the pion exchange processes dominate the cross sections in the entire energy region. The contribution of the ρ meson exchange is almost negligible at lower beam energies. Even in the energy range of 1 - 2 GeV its

contribution is less than 10 %. This is an important observation as it implies that in the description of the $A(p, \pi)B$ reaction at beam energies around 1 GeV within the covariant two nucleon model, it suffices to consider only pion exchange intermediate states, which reduces the intricacy of the calculations quite a bit [10]. The exchange of σ and ω mesons is important for beam energies closer to the pion production threshold, while at intermediate and higher beam energies their contributions are of the same order of magnitude as that of the ρ meson exchange. Therefore, it would be important to include the exchange of both σ and ω mesons in order to explain the near threshold data on pion production in pp collisions taken recently at the Bloomington cooler cyclotron [8]. It should be noted that a fully covariant calculation is necessary at even these energies because the non-relativistic reduction of the πNN Lagrangian has ambiguities, which cast doubt on the results obtained within such approaches[10, 40].

The excitation and decay of the delta isobar dominates the pion production processes at $E_{beam} \geq 0.5 GeV$ where it accounts for almost entire cross sections on its own. However, this process is not important for lower beam energies; it contributes almost negligibly to pion production near the threshold.

We have also reviewed the derivation of the covariant delta isobar propagator and have showed that the propagator given by Williams does not represent the correct propagator for a massive spin-3/2 field. This point was further stressed by performing numerical calculations with this propagator. We have found that the Williams propagator leads to very large cross sections (which look almost like a pole) for total as well as differential cross sections in certain energy regions which is clearly unphysical.

The calculations performed with a pseudoscalar coupling for the πNN vertex were found to produce too large cross sections particularly near the pion production threshold. This is due to the fact that pseudoscalar coupling alone

(without any $\sigma\pi$ coupling) produces a much too large S -wave πN interaction, which dominates the pion production near the threshold. However, this is clearly not in agreement with the experimental data. The pseudoscalar coupling is not consistent with the constraints of the PCAC. Therefore, we have preferred to use the pseudovector couplings in our calculations.

We have also analysed the available double and triple differential cross sections for pion production and found that our model provides a reasonable description of these data as well. The triple differential cross sections are sensitive to the form of the momentum dependent delta decay widths and probably also to the form of the cut-off parameters. As there are many versions of this width available in the literature, the complete kinematical detection of three particles in the final channel may perhaps help in making distinction between them. This would lead to a better understanding of the off-shell behaviour of the delta isobar [41].

In this work we established a covariant framework to describe the inelastic channels of the NN collisions. and used this to investigated the strongest of them, the pion emission. This should now form the basis for the study of relatively weaker inelastic processes like emission of η mesons and also the η' meson which is thought to provide a novel tool to study the baryonic resonances around 2 GeV [42]

Appendix A

By using the complicated but straight-forward algebra of γ matrices and the Dirac equation for a free particle the amplitudes given in subsection (2.3) can be rewritten into relatively simple forms which are suitable for numerical calculations. We have dropped the isospin factors in the expressions shown below, however in actual calculations they are included.

(i) Nucleon intermediate state and the pion exchange

$$A_N^\pi(c) = -\left(\frac{f}{m_\pi}\right)^3 \frac{1}{q^2 - m_\pi^2} \frac{1}{p_i^2 - m_N^2} \bar{\psi}(p_3) \gamma_5 \psi(p_1) \times \bar{\psi}(p_4) (E + F \gamma_\mu p_\pi^\mu) \psi(p_2), \quad (31)$$

where

$$E = 2m_N(m_N^2 - p_i^2) \quad (32)$$

$$F = p_i^2 + 3m_N^2. \quad (33)$$

It may be noted that if the pseudoscalar coupling for the πNN vertex is used, we get the same expression for this amplitude but the constants E and F are defined in the following way

$$E = 0 \quad (34)$$

$$F = 4m_N^2. \quad (35)$$

(ii) Delta isobar intermediate state and the pion exchange

$$A_\Delta^\pi(c) = -2m_N \left(\frac{f}{m_\pi}\right) \left(\frac{f^*}{m_\pi}\right)^2 \frac{1}{q^2 - m_\pi^2} \frac{1}{p_i^2 - m_\Delta^2} \times \bar{\psi}(p_3) \gamma_5 \psi(p_1) \bar{\psi}(p_4) (G + H \gamma_\mu p_\pi^\mu) \psi(p_2). \quad (36)$$

In Eq. (36) the expressions of G and H will depend on the form of the delta isobar propagator used in the derivations. For the propagator $G_{\mu\nu}^\Delta$ (Eq. (14)), one gets

$$\begin{aligned}
G &= m_N q p_\pi - \frac{2}{3} m_N p_i p_\pi + \frac{1}{3} m_N m_\pi^2 - \frac{2 p_i p_\pi p_i q}{3 m_\Delta^2} m_N + \frac{2 (p p_\pi)^2}{3 m_\Delta} \\
&\quad - \frac{p_i p_\pi m_\pi^2}{3 m_\Delta} + \frac{p_i q m_\pi^2}{3 m_\Delta} + m_\Delta q p_\pi - \frac{2}{3} m_\Delta p_i p_\pi + \frac{1}{3} m_\Delta m_\pi^2 \\
&\quad - \frac{2 p_i p_\pi p_i q}{3 m_\Delta},
\end{aligned} \tag{37}$$

$$\begin{aligned}
H &= q p_\pi - \frac{2}{3} m_N^2 + \frac{1}{3} m_\pi^2 - \frac{2 p_i p_\pi p_i q}{3 m_\Delta^2} - \frac{p p_\pi m_N}{3 m_\Delta} \\
&\quad - \frac{p_i q m_N}{3 m_\Delta} + \frac{2}{3} m_\Delta m_N + \frac{1}{3} p_i p_\pi - \frac{1}{3} p_i q.
\end{aligned} \tag{38}$$

Whereas with the propagator $G_{\mu\nu}^{\Delta W}$ (see appendix B), we obtain

$$\begin{aligned}
G &= m_N q p_\pi - \frac{2}{3} m_N p_i p_\pi + \frac{1}{3} m_N m_\pi^2 - \frac{2 p_i p_\pi p_i q}{3 p_i^2} m_N + \frac{2 (p p_\pi)^2}{3 p_i^2} m_\Delta \\
&\quad - \frac{p_i p_\pi m_\pi^2}{3 p_i^2} m_\Delta + \frac{p_i q m_\pi^2}{3 p_i^2} m_\Delta + m_\Delta q p_\pi - \frac{2}{3} m_\Delta p_i p_\pi + \frac{1}{3} m_\Delta m_\pi^2 \\
&\quad - \frac{2 p_i p_\pi p_i q}{3 p_i^2} m_\Delta,
\end{aligned} \tag{39}$$

$$\begin{aligned}
H &= q p_\pi - \frac{2}{3} m_N^2 + \frac{1}{3} m_\pi^2 - \frac{2 p_i p_\pi p_i q}{3 p_i^2} - \frac{p p_\pi m_N}{3 p_i^2} m_\Delta \\
&\quad - \frac{p_i q m_N}{3 p_i^2} m_\Delta + \frac{2}{3} m_\Delta m_N + \frac{1}{3} p_i p_\pi - \frac{1}{3} p_i q.
\end{aligned} \tag{40}$$

(iii) Nucleon excitation and ρ meson exchange

We make use of the following relations

$$q_\mu \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) = 0, \tag{41}$$

$$q^\mu \sigma_{\mu\nu} q^\nu = 0, \tag{42}$$

$$\bar{\psi}(p_3) \gamma_\mu \psi(p_1) (p_3^\mu - p_1^\mu) = 0, \tag{43}$$

to reduce the amplitude given by Eq. (22) as follows

$$A_N^\rho(c) = g_\rho^2 \left(\frac{f}{m_\pi} \right) \frac{1}{q^2 - m_\rho^2} \frac{1}{p_1^2 - m_N^2} \psi(p_4) (D_1 - D_2) \psi(p_1). \tag{44}$$

In Eq. (44) D_1 and D_2 are defined as

$$D_1 = (\gamma_\mu p_\pi^\mu \gamma_\nu p_i^\nu \gamma_\eta b^\eta - m_N \gamma_\mu p_\pi^\mu \gamma_\eta b^\eta) \gamma_5, \tag{45}$$

$$\begin{aligned}
D_2 = & - \left(\frac{k_\rho}{4m_N} \right) [-m_\pi^2 \gamma_\mu q^\mu \gamma_\nu b^\nu \gamma_5 - m_N \gamma_\mu p_\pi^\mu \gamma_\nu q^\nu \gamma_\eta b^\eta \gamma_5 \\
& + 2p_i p_\pi \gamma_\mu q^\mu \gamma_\nu b^\nu \gamma_5 - m_N \gamma_\mu p_\pi^\mu \gamma_\nu q^\nu \gamma_\eta b^\eta \gamma_5 \\
& + m_\pi^2 \gamma_\mu b^\mu \gamma_\nu q^\nu \gamma_5 + m_N \gamma_\mu p_\pi^\mu \gamma_\nu b^\nu \gamma_\eta q^\eta \gamma_5 \\
& - 2p_i p_\pi \gamma_\mu b^\mu \gamma_\nu q^\nu \gamma_5 + m_N \gamma_\mu p_\pi^\mu \gamma_\nu b^\nu \gamma_\eta q^\eta \gamma_5],
\end{aligned} \tag{46}$$

where

$$b_\mu = \bar{\psi}(p_3) [(1 - k_\rho) \gamma_\mu + \frac{k_\rho}{m_N} (p_{3\mu} - q p_{3q\mu})] \psi(p_1). \tag{47}$$

(iv) Delta isobar excitation and ρ meson exchange

Using Eqs. (41) - (43) we can write the amplitude given by Eq. (25) as

$$\begin{aligned}
A_\Delta^\rho(c) = & -g_\rho \frac{g_\rho}{m_\Delta + m_N} \left(\frac{f_\pi^*}{m_\pi} \right) \bar{\psi}(p_3) [(1 - k_\rho) \gamma^\nu + \frac{k_\rho}{m_N} p_3^\nu] \psi(p_1) \\
& \times \psi(p_4) (p_\pi^s G_{st}^\Delta \gamma_\mu q^\mu \gamma_5 - p_\pi^s G_{st}^\Delta q^t \gamma_\nu \gamma_5).
\end{aligned} \tag{48}$$

This equation can be further simplified by using the forms of the propagator G_{st}^Δ as discussed earlier. These expressions are not being given here as they are very lengthy even though it is straightforward to derive them.

(v) Nucleon excitation and σ meson exchange

$$\begin{aligned}
A_N^\sigma(c) = & \left(\frac{f_\pi}{m_\pi} \right) g_\sigma^2 \frac{1}{q^2 - m_\pi^2} \frac{1}{p_i^2 - m_N^2} \bar{\psi}(p_3) \psi(p_1) \\
& \times \bar{\psi}(p_4) (2m_N \gamma_\mu p_\pi^\mu + m_\pi^2 - 2p_i p_\pi) \gamma_5 \psi(p_2).
\end{aligned} \tag{49}$$

(vi) Pion emission from the decay of ρ meson in the intermediate states

$$\begin{aligned}
A^{\rho\pi\pi} = & i2m_N g_\rho g_{\rho\pi\pi} \left(\frac{f_\pi}{m_\pi} \right) \frac{1}{q^2 - m_\rho^2} \frac{1}{q'^2 - m_\pi^2} \\
& \times \bar{\psi}(p_3) [(1 - k_\rho) \gamma_\mu p_\pi^\mu + \frac{k_\rho}{m_N} (p_3 p_\pi - \frac{p_3 q}{q^2} q p_\pi)] \psi(p_1) \\
& \times \bar{\psi}(p_4) \gamma_5 \psi(p_2).
\end{aligned} \tag{50}$$

(vii) Nucleon intermediate state and the exchange of heavy axial meson

$$\begin{aligned}
A_N^A(c) &= g_A \left(\frac{f}{m_\pi} \right) \frac{1}{q^2 - m_A^2} \frac{1}{p_i^2 - m_\pi^2} (2p_4 p_\pi + m_\pi^2) \\
&\times \bar{\psi}(p_4) \gamma^\mu b_\mu - 2m_\pi \bar{\psi}(p_4) \gamma^\nu p_{\pi\nu} \gamma^\mu b_\mu \psi(p_2),
\end{aligned} \tag{51}$$

where

$$b_\mu = \bar{\psi}(p_3) \gamma_5 \gamma_\mu \psi(p_1) \tag{52}$$

The Numerical evaluation of the amplitudes written in the forms given above can be carried out very effectively by using the techniques of the Clifford algebra.

Appendix B

In this appendix we present the discussion on the form of the delta isobar propagator.

The free Lagrangian density for the massive spin-3/2 field is written as [43]

$$\mathcal{L}_\Delta = \bar{\Psi}^\mu \Lambda_{\mu\nu} \Psi^\nu, \tag{53}$$

where the most general form of $\Lambda_{\mu\nu}$ is given by [43]

$$\begin{aligned}
\Lambda_{\mu\nu} &= -[(-i\partial_\mu \gamma^\mu + m_\Delta) g_{\mu\nu} - iZ_1(\gamma_\mu \partial_\nu + \partial_\mu \gamma_\nu) \\
&\quad - Z_2 \gamma_\mu (\partial_\lambda \gamma^\lambda) \gamma_\nu - Z_3 m_\Delta \gamma_\mu \gamma_\nu]
\end{aligned} \tag{54}$$

Here Z_1 is an arbitrary parameter subject to the restriction that $Z_1 \neq -1/2$, and Z_2 and Z_3 are defined as

$$Z_2 = \frac{1}{2}(3Z_1^2 + 2Z_1 + 1), \quad Z_3 = (3Z_1^2 + 3Z_1 + 1) \tag{55}$$

Physical properties of the free field do not depend on the parameter Z_1 , which is chosen to be real. This is due to the fact that \mathcal{L}_Δ is invariant under the point transformation [43]

$$\Psi^\mu \rightarrow \Psi^\mu + a \gamma^\mu \gamma^\nu \Psi_\nu \tag{56}$$

$$A \rightarrow \frac{Z_1 - 2a}{1 + 4a}, \tag{57}$$

where $a \neq -\frac{1}{4}$, but is otherwise arbitrary.

The wave equation for the spin-3/2 particle (the Rarita-Schwinger) equation is written as

$$\Lambda_{\mu\nu}(p)\Psi^\nu = 0 \quad (58)$$

Operating on Eq. (58) with γ_μ and ∂^μ we get the local wave equation for a spin-3/2 particle along with the constraint equations

$$(i\partial_\nu - m_\Delta)\Psi^\mu = 0 \quad (59)$$

$$\gamma_\mu\Psi^\mu = 0 \quad (60)$$

$$\partial_\mu\Psi^\mu = 0 \quad (61)$$

It should be noted that Eqs. (59) - (61) are obtained only when the restriction $Z_1 \neq -1/2$ and the definitions of Z_2 and Z_3 as given in Eq. (55) are used.

The propagator for a massive spin-3/2 particle satisfies the following equation in the momentum space

$$\Lambda_{\mu\nu}(p)G_{\Delta\alpha}^\nu = -g_{\mu\alpha} \quad (62)$$

Solving for G and making the particular choice of $Z_1 = -1$ we get the following equation for the delta isobar propagator

$$G_{\mu\nu}^\Delta(p) = -\frac{i(\not{p} + m_\Delta)}{p^2 - m_\Delta^2} \left[g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2}{3m_\Delta^2}p_\mu p_\nu + \frac{1}{3m_\Delta^2}(p_\mu\gamma_\nu - p_\nu\gamma_\mu) \right] \quad (63)$$

A slightly different form of the Δ propagator has been suggested by Williams [20], which is given by

$$G_{\mu\nu}^{\Delta W}(p) = -\frac{i(\not{p} + m_\Delta)}{p^2 - m_\Delta^2} \left[g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2}{3p^2}p_\mu p_\nu + \frac{1}{3p^2}p_\mu\gamma^\eta(p_\nu\gamma_\eta - p_\nu\gamma_\mu) \right] \quad (64)$$

Williams has argued that the Eq. (64) should be the valid form of the Δ isobar propagator both on and off the mass shell. However, we show in the following that it can not be the correct spin-3/2 propagator.

First, we note that if $G_{\mu\nu}^{\Delta W}$ is the propagator for a spin-3/2 particle, then it should satisfy

$$\Lambda'^{\mu\nu}(p)G_{\nu}^{\alpha\Delta W} = -g^{\mu\alpha}, \quad (65)$$

where Λ' is defined as

$$\Lambda'_{\mu\nu} = -[(-i\partial_{\mu}\gamma^{\mu} + m_{\Delta})g_{\mu\nu} + i\lambda\gamma_{\mu}\partial_{\nu} - i\lambda\partial_{\mu}\gamma_{\nu}], \quad (66)$$

with the limit that $\lambda \rightarrow \infty$ (known as Feynman gauge). However, Eq. (66) is not consistent with the general form of the Lagrange function of a spin-3/2 particle as given by Eqs. (54) and (55).

Second, in Ref. [22], it has been shown that the propagator $G_{\mu\nu}^{\Delta W}$ has no inverse, and thus it can not be the propagator of a spin-3/2 particle. A similar difficulty is associated with the propagator suggested by Adelseck et al. [21].

In Fig. 9a and 9b, we study the effect of using the two delta isobar propagators $G_{\mu\nu}^{\Delta}$ and $G_{\mu\nu}^{\Delta W}$ in the calculations of the total and triple differential cross sections for the $p(p, n\pi^+)p$ reaction to stress further the difficulties associated with the Williams propagator. The solid lines show the results obtained by using the former propagator while the dashed line the later one. It is clear that the $G_{\mu\nu}^{\Delta W}$ produces very large cross sections in a certain range of beam energies due to the fact that the factor p_i^2 present in the demoninators of some terms of this propagator, becomes very small. This effect is particularly very strong for the pre-emission graphs. For example, for such a graph which is analogous to Fig. 1a we have

$$\begin{aligned} p_i^2 &= (p_2^2 - p_{\pi}^2) \\ &= m_N^2 - 2E_{\pi}m_N + m_{\pi}^2 \end{aligned} \quad (67)$$

It can be seen that for $E_{\pi} \simeq 0.479 \text{ GeV}$, p_i is very close to zero. This energy corresponds to an incident proton energy of approximatly of 1.2 GeV . Therefore,

at incident energies around this value the terms which are proportional to $(1/p_i^2)$ in $G_{\mu\nu}^{\Delta W}$ become very large in the pre-emission graphs which in turn leads to huge total cross sections. In the post-emission graphs this problem is not so severe. This very clearly shows the difficulty that one encounters while using the propagator $G_{\mu\nu}^{\Delta W}$. It must be mentioned here that normally the contributions of pre-emission graphs are much smaller than those of the post-emission ones if one uses the propagator $G_{\mu\nu}^{\Delta}$.

The effect of using $G_{\mu\nu}^{\Delta W}$ is very drastic in case of triple differential cross sections as can be seen in Fig. 9b. Williams propagator leads to a very large cross section (which looks almost like a pole) at certain value of the outgoing pion momenta due to the same reason as discussed above.

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Table 1: **Coupling constants for the NN meson vertices used in the calculations**

| Meson | $g^2/4\pi$ | ℓ | Λ (GeV) | mass (GeV) |
|--|------------|--------|---------------------|---------------|
| π | 12.562 | 0.1133 | 1.005 | 0.138 |
| σ | 2.340 | 0.1070 | 1.952 | 0.550 |
| ω | 46.035 | 0.0985 | 0.984 | 0.783 |
| ρ | 0.317 | 0.1800 | 1.607 | 0.770 |
| $k_\rho = 6.033, k_\omega = 0.0, g_{\rho\pi\pi} = 2g_\rho$ | | | | |

Table 2: Isospinfactors for pole diagramms

| nucleon pole | | | delta pole | | |
|--------------------------|-------------|------------|--------------------------|---------------|-----------|
| $pp \rightarrow np\pi^+$ | | | $pp \rightarrow np\pi^+$ | | |
| graph | isovector | isoscalar | graph | isovector | isoscalar |
| a | $\sqrt{2}$ | $\sqrt{2}$ | a | $-\sqrt{2}/3$ | 0 |
| b | 0 | 0 | b | $\sqrt{2}$ | 0 |
| c | 0 | 0 | c | $\sqrt{2}$ | 0 |
| d | $\sqrt{2}$ | $\sqrt{2}$ | d | $-\sqrt{2}/3$ | 0 |
| pre a | $-\sqrt{2}$ | $\sqrt{2}$ | pre a | $\sqrt{2}/3$ | 0 |
| pre b | $2\sqrt{2}$ | 0 | pre b | $\sqrt{2}/3$ | 0 |
| pre c | $2\sqrt{2}$ | 0 | pre c | $\sqrt{2}/3$ | 0 |
| pre d | $-\sqrt{2}$ | $\sqrt{2}$ | pre d | $\sqrt{2}/3$ | 0 |
| $pp \rightarrow pp\pi^o$ | | | $pp \rightarrow pp\pi^o$ | | |
| all graphs | | 1 | all graphs | | 2/3 |

Table 3: Isospinfactor for direct diagramms

| intermediate | |
|--------------------------|--------------|
| $pp \rightarrow np\pi^+$ | |
| graph | |
| e | $-i\sqrt{2}$ |
| f | $i\sqrt{2}$ |
| g | $i\sqrt{2}$ |
| h | $-i\sqrt{2}$ |
| $pp \rightarrow pp\pi^o$ | |
| all graphs | 0 |

Figure Captions

Fig. 1 . Feynman diagrams for emission of a pion in the nucleon - nucleon collisions. (a) Exchanged meson starts from nucleon 2 (momentum p_2), is absorbed by nucleon 1 (momentum p_1) which is excited to the nucleon or delta isobar intermediate state with momentum (p_i) which then decays into the nucleon 3 (momentum p_3) and the outgoing pion (momentum p_π). The nucleon 2 goes on to nucleon 4 with momentum (p_4). (b) Exchange part of the diagram (a). (c) Same as (a) but the exchanged meson starts from nucleon 1 and is captured by nucleon 2 which is excited to intermediate states which then decay into nucleon 4 and the outgoing pion. Nucleon 1 goes on to become nucleon 3. (d) Exchange part of diagramme (c). (e)-(h) Direct and exchange diagrams showing the processes where the exchanged ρ meson starting from one of the interacting nucleons decays into two pions in flight. One of them is the outgoing pion, the another one is absorbed by the other nucleon. These diagrams are referred as intermediate graphs in the text. Note that there are also the pre-emission counter-parts of the diagrams (a) - (d) where pion are emitted before collisions. These graphs are not shown here, but their contributions are included in the calculations.

Fig.2a . The total cross section for the $p(p, n\pi^+)p$ reactions as a function of beam energy. The dashed and dashed-dotted lines represent the results of the calculations performed with only delta isobar intermediate states and only nucleon intermediate states, respectively. The sum of all the graphs is represented by the solid curve. The experimental data are taken from [3].

Fig.2b . Same as Fig. 2a but for the reaction $p(p, p\pi^0)p$.

Fig. 3 . Contributions of various exchanged mesons to the total cross section for the reaction $p(p, p\pi^+)p$ as a function of beam energy. The long dashed,

dotted, short dashed and dashed-dotted curves represent the contributions of π exchange, ρ exchange, ω exchange and σ exchange, respectively.

Fig. 4 . Total cross sections for the reaction $p(p, p\pi^+)p$ as a function of beam energy for two values of the cut-off parameter of the πNN vertex. The solid (long dashed) line is the result of calculations performed with a value of 1.005 GeV (0.631 GeV) for this parameter.

Fig. 5 . Ratio of the total cross sections calculated with pseudovector and pseudoscalar couplings for the πNN vertex for the same reaction as in Fig. 4, as a function of beam energy. The long dashed line represents the results obtained with only nucleon intermediate states while the solid line contain all the graphs.

Fig. 6 . The double differential cross section for the reaction $pp \rightarrow \pi^+ + X$ as a function of pion momentum for pion angles of 20° , 40° and 60° at a beam energy of 800 MeV. The dashed (dotted) line represents the results obtained with only delta isobar (only nucleon) intermediate states. The solid line shows the results obtained by including all the graphs.

Fig. 7 . The triple differential cross sections for the $p(p, n\pi^+)$ reaction at the beam energy of 800 MeV as a function of the outgoing pion momentum. The upper part shows the results for the proton and pion angles of 15° and 21° respectively while the lower part for 25° and 40° respectively. The dashed lines show the results when only delta isobar intermediate states are included in the calculations while the solid lines depict the results obtained by including all the graphs.

Fig. 8 . The effect of using various forms of delta decay width in the calculation of triple differential cross sections for the same reaction as in Fig. 7 and

at the same beam energy. The solid and dashed lines represent the results obtained with the forms of the delta decay widths given by Verwest and Dmitriev with a free delta decay widths of 120 MeV. The dotted lines represent the results obtained with the later but with a a free delta decay width of 100 MeV. The results are shown as a function of outgoing pion momentum.

Fig. 9a . Total cross section for the $p(p, p\pi^+)p$ reaction as a function of beam energy calculated with the propagators of Benmerrouche et al. [22] (solid line) and Williams [20] (long dashed line).

Fig. 9b . Triple differential cross section for the $p(p, p\pi^+)p$ reaction as a function of outgoing proton momentum corresponding to proton and pion angles of 15° and 21° respectively, The solid and long dashed curves have the same meaning as in Fig. 9a.

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